

IS A SCIENTIFIC IDEA OF HOPE
POSSIBLE? HOW DOES IT RELATE TO
OUR IDEAS ABOUT THE PHILOSOPHY
OF RELIGION?

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One way hope is with the odds, expected, certain, is when it is mathematical. The first theologian to figure out the details of this, to give definition to what had yet to be defined, was the French theologian, mathematician, scientist, Blaise Pascal

- John Calvin and the Dutch Bishop Jansen believed that Hope comes about against all odds. But, the French Theologian Blaise Pascal was trying to defend Jansen against criticism from the Jesuits and the Church that they were not consistent with St. Augustine's teachings. To do this had to try and explain what "Hope with the odds" might mean. Along the way the foundations were laid for our modern understanding of what mathematical "probability" means. Do you agree with how Pascal defines probability? And what type of Hope do you believe in (against all odds or with the odds). My talk in the MAS mathematics/statistics/computer science section will concentrate on the mathematical details of how Pascal's laid the foundations for modern axiomatic probability theory. While my talk in this section will concentrate on the theology and philosophy of Pascal's theories of scientific definition and his idea of how "hope" and "mathematical expectation" might be understood using these maxims (rules of investigative thought) and methods of definition.

- The best reference to Pascal's ideas are his own writings. See Volume 33 of Enc.Brit. Great Books, compiled by our former letter correspondent and prayer friend , public philosopher, and my uncle William Calohan's teacher at U. of Chicago in the 1930s, Dr. Mortimer Adler. When I was doing my science fair project in Vicksburg, MS (and 10 or 11 years old) in the late 1950s I wrote Dr. Adler for information about the formula for relating temperature to cricket chirps. He was kind enough to answer my letter and send me 20 pages of information. 25 or so years later I had gotten a doctorate in Math and wrote him again about the mathematics and philosophy related to Zeno's paradox and Plato's Parmenides dialogue. I don't know if he remembered me but he also answered the 2nd and more advanced letter. I noticed in the 1990s his son Isaac is now also a professional mathematician working and studying algebraic invariants. He gave a series of lectures at the the UC Berkeley mathematic's institute above I-House and Strawberry Canyon, Berkeley/Oakland East Bay.

Bishop Jansen was trying to learn how to understand St. Augustine's doctrines as compared against those of the Jesuits. Is God's grace which He gives to all men, always sufficient unto itself [ie inside of all of us]? The Jesuit's did not like Jansen's theology because it followed John Calvin's to some extent. One of Calvin's five theological points was that the grace of God, once received, was sufficient and thus did not require the help of someone else, eg. a priest to enable it and make it effective. St. Augustine believed that this was impossible due to our original sin, the Jesuit's said, "maybe not" but we are against any attack on the authority of the Church structure. The "maybe not" referred to here has to do with the theology of morals and the question of whether if as Aristotle said, "those who don't know whether they are doing anything wrong actually are sinning"? ref. Great Books, volume 33[Letters III,IV,V]

- In my talk in the mathematics section I discussed how Pascal proved his key theorem that justifies the use of his arithmetical triangle to calculate probabilistic “expectations” [hopes] for the gamblers problem in the “game of points”. The solution to this problem was an important event in 17th century mathematics and allowed Pascal to create a new mathematical discipline called probability theory. The mathematician Blaise Pascal spent a good part of his life thinking about the scientific/theological question of how to define probability

- In his essay **“On Geometrical Demonstration (on the Geometrical Mind) included later in the Great Books volume Pascal explains his ideas about some fascinating theories of mathematical proof and definition.** It is interesting to compare this essay on Pascal’s mathematical method of discovery and proof with Descartes, and later Poincare’s. But, that is a more difficult discussion. I will just mention some of his aphorisms and assertions here:

“We must give definition to the defined. [in order to discovery new truths]”

Rules for definition:

- 1) do not attempt to define any of those things so well known in themselves that we have no clearer terms to explain them by.
- 2) Do not leave undefined any terms that are at all obscure or ambiguous
- 3) Use in the definitions of terms only words perfectly well known or already explained.

Rules for axioms:

- 1) Do not fail to ask that each of the necessary principles be granted, however clear and evident it may be.
- 2) Ask only that perfectly self-evident things be granted as axioms
- 3) “Time is [defined as] the motion of a created thing”

“The three fundamental concepts of science and mathematics are:

- 1) motion
- 2) number *
- 3) space

*in relation to understand how to define number you might want to see our discussions on “How do we define the number one?” that are posted on the <http://www.yhwhschofchrist.org/discussionboard> Science and Religion directory. It focuses on the German Logician and Mathematician Gottfried Frege’s ideas of this, the history of the development of the modern computer, and how this question relates to epistemology and the question “What is the Concept of a Concept”.

If we say we “probably don’t know” what do we mean? Apparently, this started Pascal thinking about the question what do we mean when we use the word “probably”? How is hope defined differently than we define faith? Faith has to do with a reality in space and time that we find ourselves and so does hope. But, what is the difference between them? In general hope is usually more connected with natural causation and faith with subjective points of view. Depending on whether we say that mathematics is about reality (like Plato) or not (like some more modern philosophers) mathematics is connected to how we understand reality in space and time (in terms of natural causes) and how we understand logical truth which can exist as a hope or faith outside of space and time. When talking about the theological arguments of some of the Jesuits when they tried to discredit the Protestant theologians of the time, Pascal says: “[It is] a very singular thing to save one man’s suffering, by imposing it on another”, he says [Letter VIII].

He sets forth three rules for ethics, 1) “the spirit of piety always prompts us to speak with sincerity and truthfulness; whereas malice and envy make use of falsehood and calamity”, 2) “none may do the least evil in order to accomplish the greatest good” for “the truth of God stands in need of no lie”, 3) but it is not enough just to tell nothing but the truth, but “we must not speak that which can hurt, without doing any good”. He quotes Saint Augustine, “the wicked in persecuting the good, blindly follow the dictates of their passions, but the good are guided by a wise discretion.” And, he quotes Jesus’s words, “woe to the blind leaders, woe to the blind followers” against these arguments [Letter XI]. Even though He already knows the answer [who we are and where we are coming] God cannot help us until we tell Him the truth about it.

The mathematician Blaise Pascal spent a good part of his life thinking about the scientific/theological question of how to define probability. Pascal got started as a theologian by defending the 17th century Dutch bishop Cornelius Jansen [Letters I and II].

- **The game of points studied by Fermat and Pascal consisted of a stake being made by both players, then a fair coin being flipped a number of times.**
- The stake goes to whichever player has a given number of heads or the most heads at the end of the flipping. If, at a certain point in the game [before the given number of points has been achieved by either player] they decide to quit and divide up winnings so far, the question comes up, “how is this to be done?”. Thus the game is an example of what is called nowadays a Bernoulli trial [the probability of the results of each flip are independent of the results that have gone before]. It is clear from the fact that there are only two possibilities for each throw that a given game of n points can only last for at most $2n$ throws. And, if, at a certain point in game it m out of $2n$ throws have already occurred, then in order to figure out the solution to the problem as stated, it is only necessary to consider the possibilities and combinations of throws out to $2^{(n-m)}$ more throws.
- After Fermat’s letter [see Great Books volume 33 reference] Pascal started thinking about if the game was analyzed during its course of play how to we define the probability for one player winning, considered what has happened up to that time.
- **In order to do this he formulated a tentative definition, later to be made more precise by Laplace:** “The theory of chance consists in reducing all the events of the same kind to a certain number of cases equally possible, that is to say, to such as we may be equally undecided about in regard to their existence, and in determining the number of cases favorable to the event whose probability is sought. The ratio of this number to that of all the cases possible is the measure of this probability, which is thus simply a fraction whose numerator is the number of favorable cases and whose denominator is the number of all the cases possible.” — Pierre-Simon Laplace,
- A Philosophical Essay on Probabilities

- Certain rules were postulated around this time by mathematicians studying in this area (or assumed) as to how to calculate “probabilities $p(E)$ of an event E ”
- 1) for all events $0 < p(E) < 1$
- 2) $p(\text{impossible events}) = 0$. $p(\text{certain events}) = 1$
- 3) $p(\text{ not an event happening}) = 1 - p(E)$
- 4) If two events, A, B are disjoint in occurrence: $p(A \text{ or } B) = p(A) + p(B)$
- 5) If two events A, B are with independently determined outcomes, but successive results of the same experiments $p(A \text{ then } B) = p(A \text{ and } B) = p(A) * p(B)$
In order to determine the ratio of favorable possible outcomes to unfavorable at a particular point in the analysis of the partial results of the complete results

- **Pascal computed a lot of the values of what we now call, due to Newton I believe, binomial coefficients** [because they are the coefficients of the expansion of the powers a binomial function $(a + b)^n$ [(a+b) raised to the power n] = sum of terms $k=1$ to n $\binom{n}{k} a^k b^{n-k}$ (. If you think about it in terms of how the powers of $a^k b^{n-k}$ you will understand that these coefficients count the number of ways of picking k objects out of a set of n (see my mathematical talk for more on this). In the course of doing this he discovered the important step by step relationship which allows us to compute a table of the values of these coefficients for a given n, assuming we already know what the coefficients are for the case of n-1.
- Pascal's algorithm to compute his famous arithmetic triangle which can be used to compute binomial coefficients First of all, by the definition of the binomial coefficients in terms of combinations of k objects picked out of n we know they have the property that the number is
- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

- **Those interested can consult the excellent Mathematical Association of America's (MAA) introductory text "Mathematics of Choice, How to Count without Counting" by Dr. Ivan Niven** and the more advanced one "Combinatorial Mathematics" by Herbert Ryser for more details on these Theorems . Also, the MAA's book, "The Mathematics of Games and Gambling" by Edward Packel has a lot of information about how to use binomial coefficients to compute the probabilities of results in backgammon, roulette and craps and also bridge and poker hands. Binomial coefficients and the arithmetic triangle have many applications. In addition to the theory of games and gambling already mentioned, then can be used to count out and construct symmetrical experimental designs, patterns of symmetry and tilings in continuous and discrete transformation groups, fractal image compression transformations (see my UC Berkeley graduate mathematics department friend Hans Otto Pietgen and his several Springer books on Fractals).

- One of the applications that Pascal gives of the arithmetical triangle that is most relevant to the topic of this talk is to compute what he defines as the "expected payoff" or "probabilistic expectation" or "mathematical expectation" at a given position in a game of one player versus the other in the game of points.
- Let $E_1, E_2, E_3, E_4 \dots E_N$ be pairwise disjoint events (no pair can occur simultaneously) with their respective probabilities $P_1, P_2, P_3, P_4 \dots P_N$. Then what we call the "mathematical expectation" in an experiment in which one of these events must occur is defined to be equal to:
 - $P_1 * E_1 + P_2 * E_2 + P_3 * E_3 + P_4 * E_4 + \dots P_N * E_N$
 - **Pascal's Theorem** [used to compute expectation values for the game of points]:
 - In a game of points
 - in which each player lacks a certain number of points r [1st player] and s [2nd player] to win:
 - In order to find the division of expectations [hopes for a win of the stakes]
 - Use the arithmetical triangle with base of order $n=r+s$ and add up the coefficients of the base of
 - The triangle of that order in that proportion [the sum of those r coefficients is compared
 - To the sum of those s]
 - The sum of all the coefficients in the base row is equal to the sum of the binomial coefficients of
 - the ways k objects can be taken from n [computed by means of the formula to be
 - equal to $\binom{n}{k} = \frac{n * (n-1) \dots (n-k+1)}{k * (k-1) \dots 1}$ where n is the number of the base
 - And k is the index running diagonally along the base.

- In order to prove the validity of the formula for the expectations of each player:
- expectation of 1st player = number of ways of winning in last $r+s$ throws/ total number of possible throws
- expectation of 2nd player = number of ways of winning in last $r+s$ throws/total number of possible throws
- we use the 4th axiom listed above which define the probability of a series of disjoint events {the throws} being the sum of the probability of each. And, we also use the 3rd axiom to tell us that to get the probability at each throw we multiply the independent particular combinations of probability of separate dice outcomes times the total number of possible outcomes.
- In order to prove the validity of the algorithm for all triangles [and hence all game situations], since we have already identified the expectation as a ratio of sums of binomial coefficients and we know the base rows in the triangle are made up of these coefficients, defined inductively and recursively by Pascal's formula (which already has been proved in two ways) the theorem follows by induction on the base index of the triangle, by using base index as the inductive index.

- Thus, considering this, the question is, how do these different approaches to computing what "hope" means mathematically translate into differences in theology?

Not an easy question, is it? Pascal noted in his essays that if we use the same reasoning as above, eg. same axioms to define probability and same rules to compute probabilistic expectation and apply it to the question of whether it is reasonable to believe that God exists and that there is some argument for holding this belief. Let the probability that God exists be $P > 0$. If we believe that God exists and He does not then there will be some negative payoff due to wasted time and energy and effort. Call this payoff $-Z$. If we believe in Him and He does exist there will be some positive payoff. Call this payoff X . If we do not believe in God and He does not exist there will be a positive payoff. Call this payoff Y .

- However, if God does exist and we do not believe in him the payoff will be a much worse, missing out on all the benefits of His eternal help, and possibility suffering damnation. If there is no God to save us and we are trying to accomplish goals over a long period of time without knowing we have all the time in the world to do it the possibility of success is much less. Call it $-\infty$. If God does exist instead of a finite time to accomplish things we have unlimited time. So the benefits in this case are infinitely positive. So, the effects of God existing and not are infinitely positive and zero respectively.

So translating all of this into an expectation formula we have two very different sums of weights times probabilities for $X(\text{belief})$ and $X(\text{nonbelief})$:

$$X(\text{belief}) = P * X + (1-P) * (-z)$$

$$X(\text{non-belief}) = P * (-\infty) + (1-P) * Y = -\infty$$

And, when we sum the expectation formulas, at any given time in our lives, as in the game of points the positive part is heavily outweighed by the effect of the negative infinity part in the second of the formulas summing up the possibilities.

- Thoughts posted on LinkedIn by Ioannis Zaglaris • SCIENTIFIC IDEA OF HOPE, by Andrew Harrell & SCIENCE AND PHILOSOPHY, by Byron Jennings (I.Z. commenting) hope is the expectation for Goodness to come. Goodness is a simultaneous motion of our soul and of our body. philosophy is the guard of our sleeping soul and science is suing the buying and selling of products until our soul awakes and assumes responsibility. Literally the idea in Greek is for a writer -painter or poet- to prove mathematically the eternal existence of this motion. therefore to prove existence of perpetuum mobile or eternity of Greek soul, (=Second Coming). 0 Andrew Harrell • Yes, John, if we define goodness as Aristotle does (what exists for its own sake), then goodness can use its spiritual self- existence inside of a changing environment in order to move its soul along with its body and its body with its soul. But, my question remains, how did Pascal's arithmetic triangle change for the better our understand of hope? My idea is that relating [mathematical proving using geometrical and logical relationships] of the definitions of combinations of things inside of each other (binomial coefficients) to the values of a sequence of probabilistic expectations [hopes for goodness to come] this happened for us all in those days at that time.

- **Also, it is all related, also, to the more fundamental and ever-present theological question of whether Jesus Christ is already alive in us right now, as individuals living on our own apart from the Church, or yet to come?** Does Jesus come inside of us all before the millennium or after it? And if not both ways, why not both ways? And on and on. The answer to this question determines how we plan and pray for heaven to exist inside of us and who if not ourselves as individuals, who is in control of God outside of us as individuals and also inside of us as unique persons, 1) It determines those who we anoint to run the Church, 2) those who we anoint to understand the Bible, or the Torah, or the Quran, or 3) and since the Church is a powerful controlling force in secular society it often determines also how to have money or power in the World we live in? Unfortunately the answer to this problem often gets lost while investigating its subtleties. Maybe to spite those people who didn't take kindly to him declaring this question to often for discussion, Pascal turned his thinking toward how we can understand each other in different ways, how we can understand the theory of gambling odds mathematically, how we can measure temperature with an instrument, and how we can understand what the vacuum is scientifically. While doing this he became one of the more important scientists and mathematicians of that important century in the history of human enlightenment.