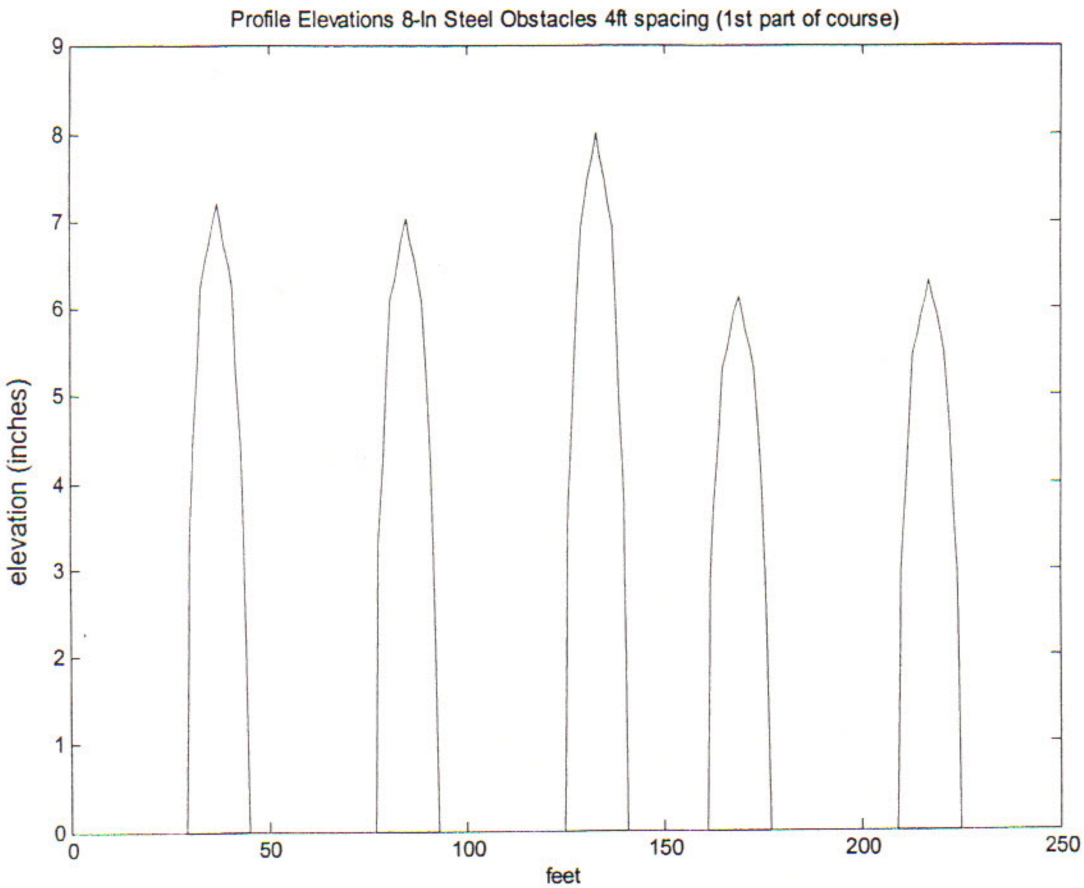

Profile Elevations 8-In Steel Obstacles 4ft spacing (1st part of course)
elevation (inches)
feet
0
1
2
3
4
5
6
7
8
9
0
50
100
150
200
250

Figure 31

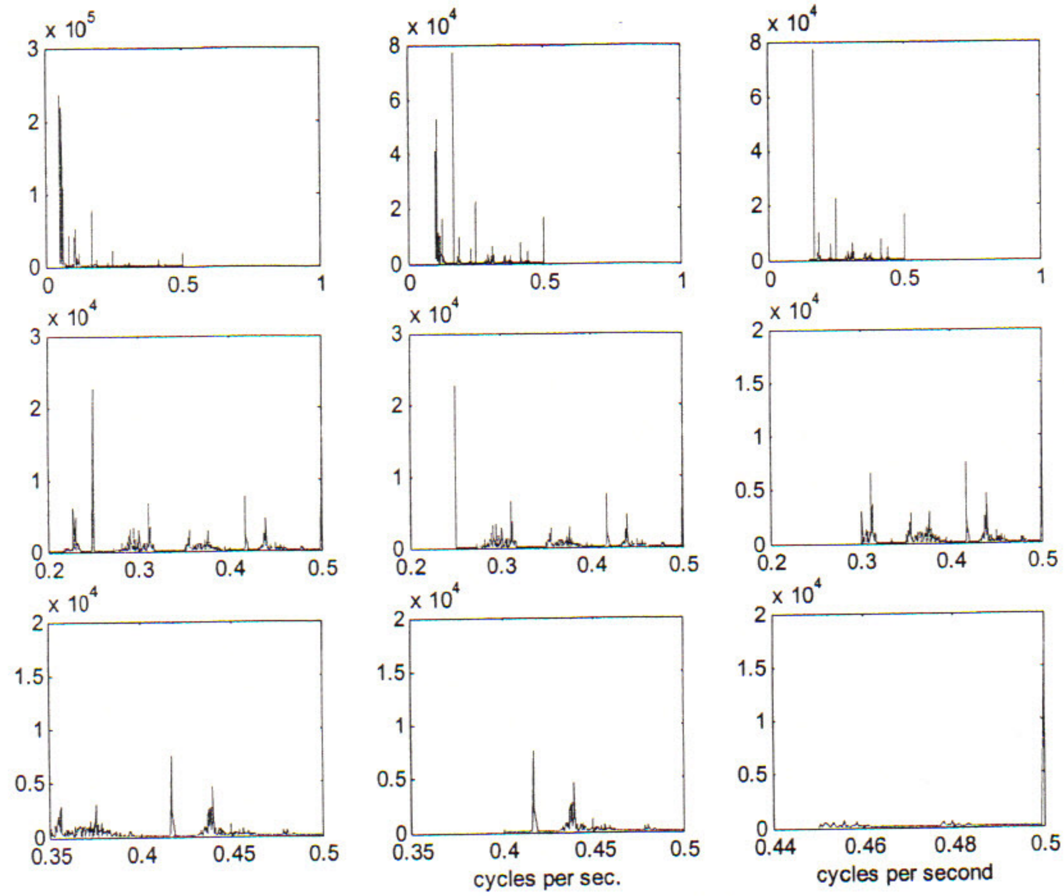


Figure 32 Scaled Power Spectrums of 8" Steel at 4' spacings

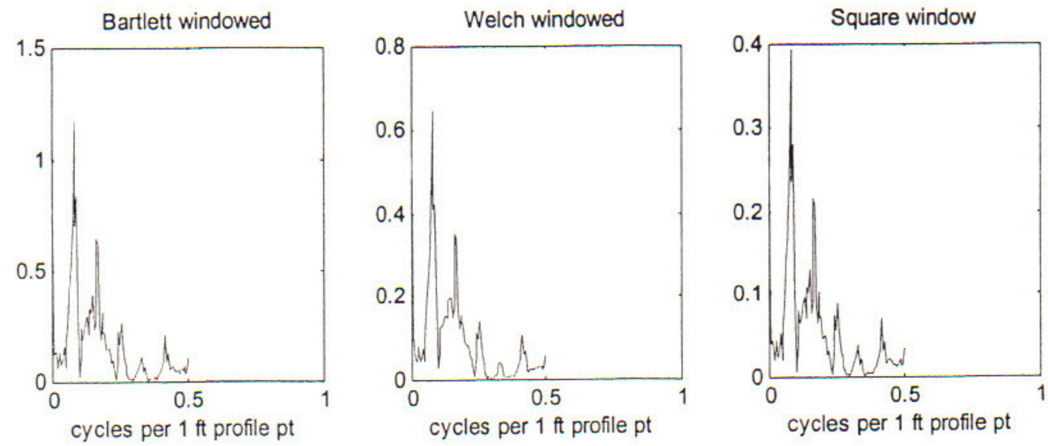


Figure 33 Windowed Power Spectrum of 8' Steel at 4 ft Spacings

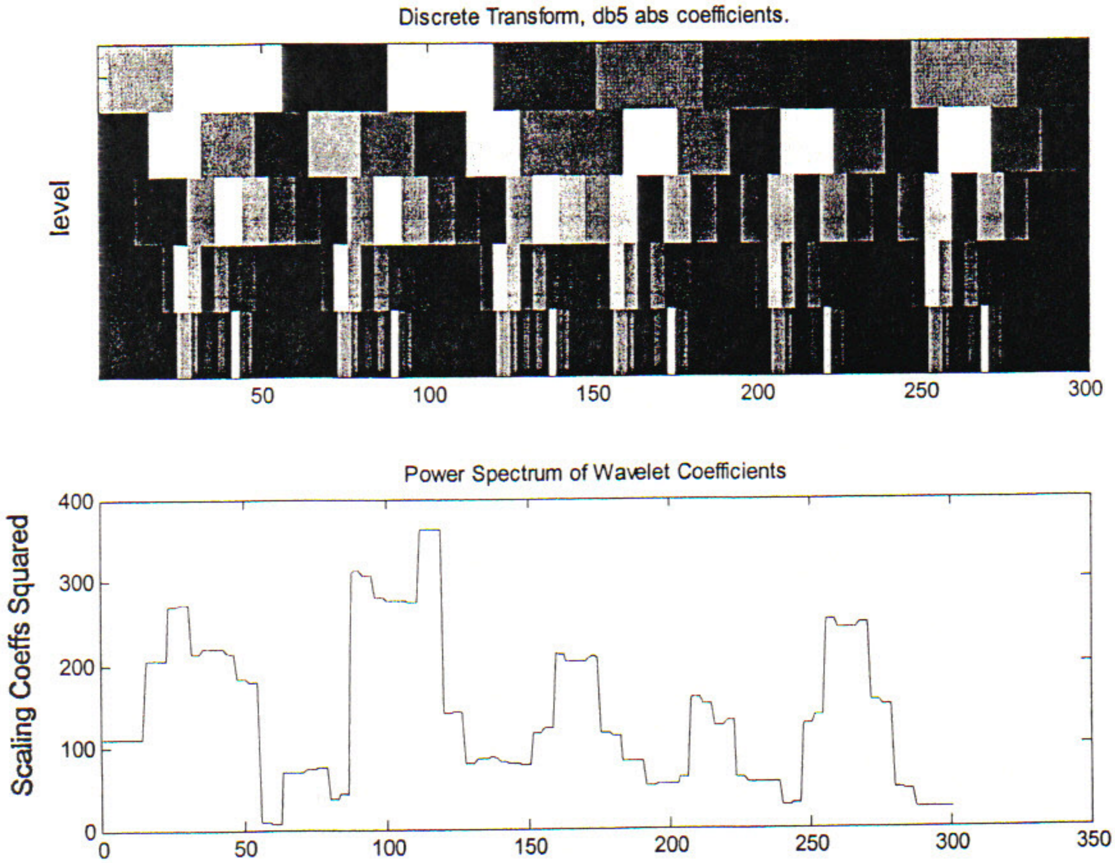


Figure 34 Wavelet Power Spectrum, db5 Coefficients

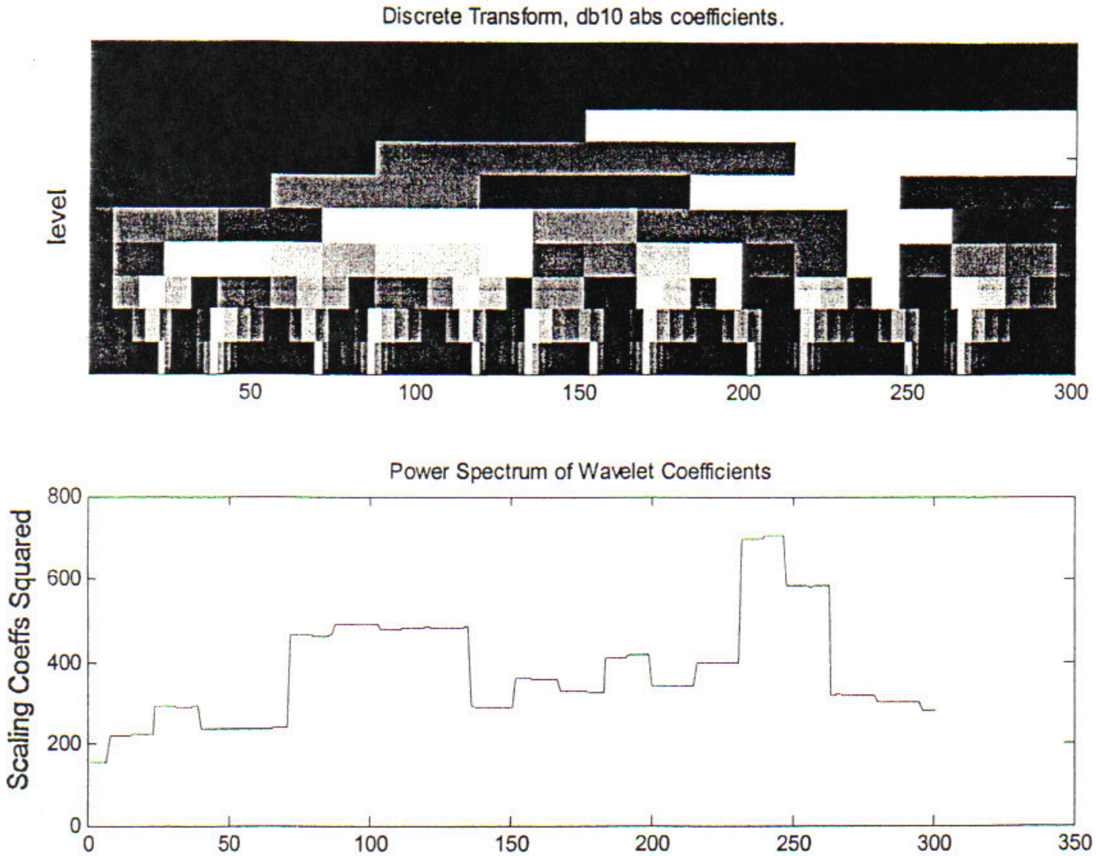
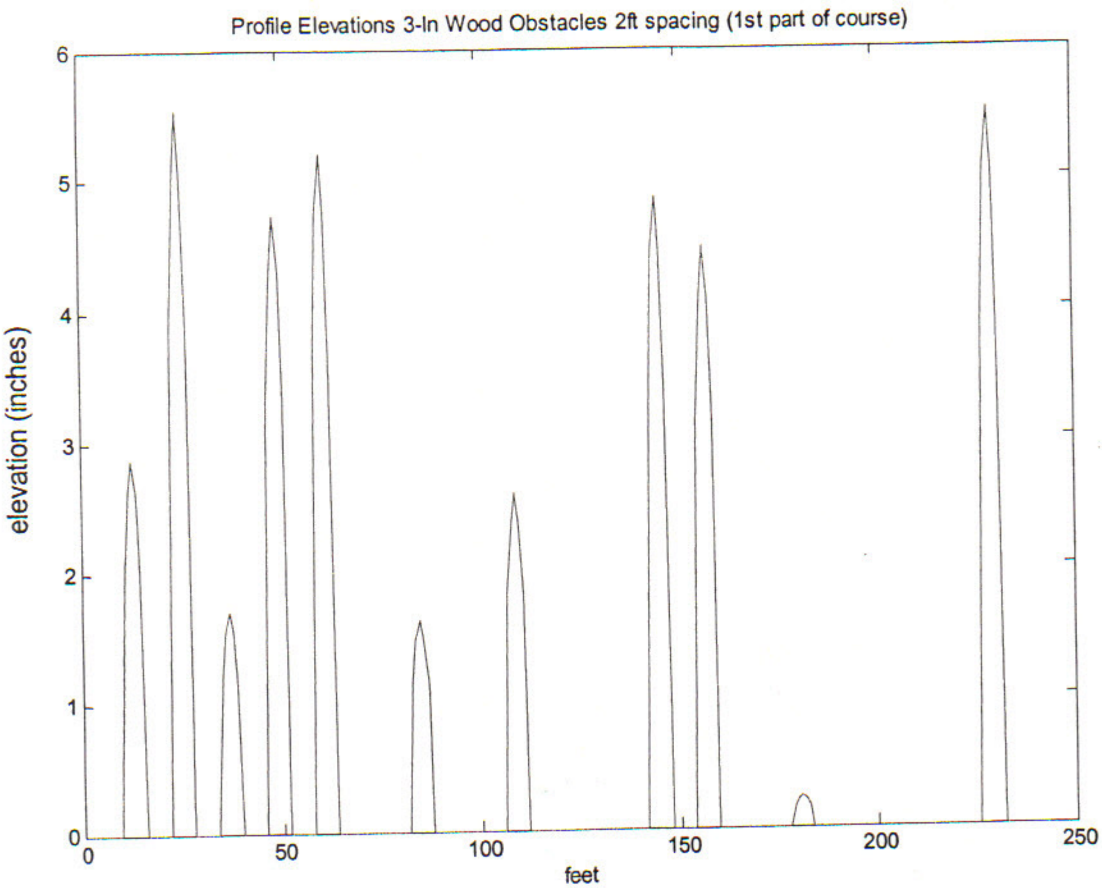


Figure 35 Wavelet Power Spectrum, db10 Coefficients



Profile Elevations 3-In Wood Obstacles 2ft spacing (1st part of course)
elevation (inches)
feet
0
1
2
3
4
5
6
0
50
100
150
200
250

Figure 36

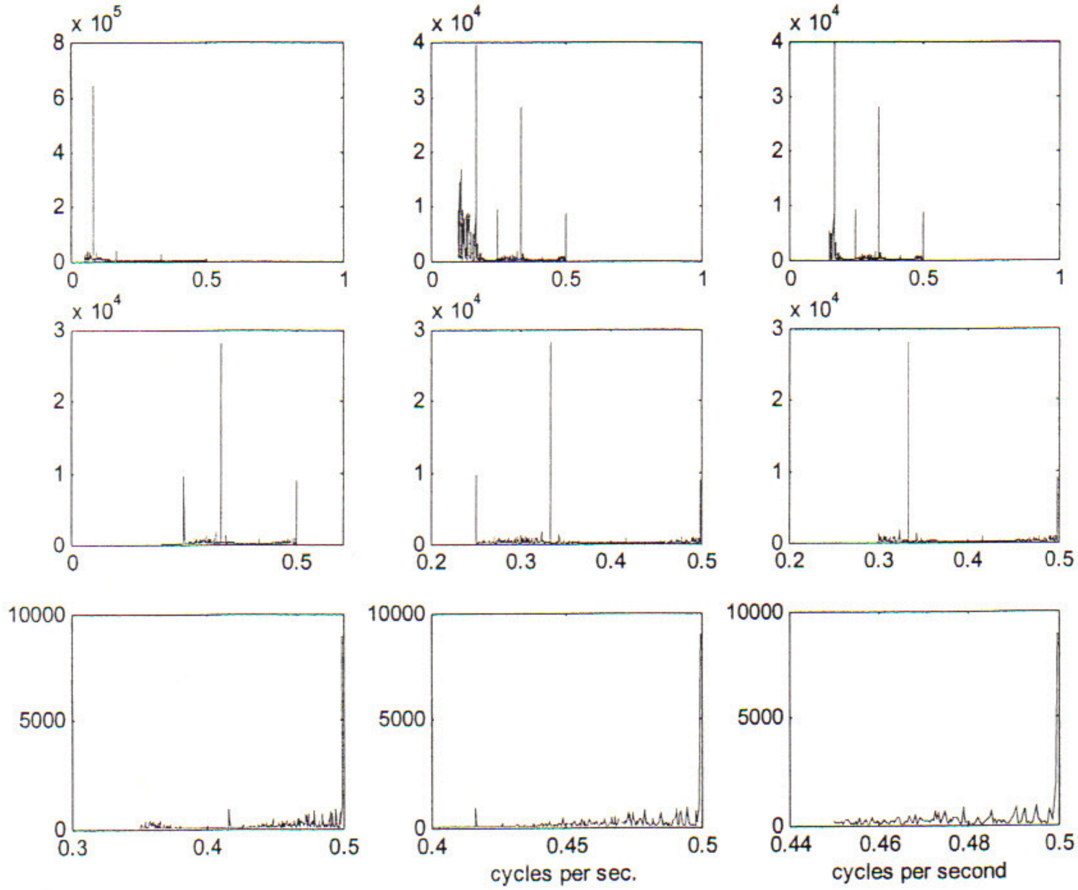


Figure 37 Scaled Power Spectrums of 3" Wood at 2' spacings

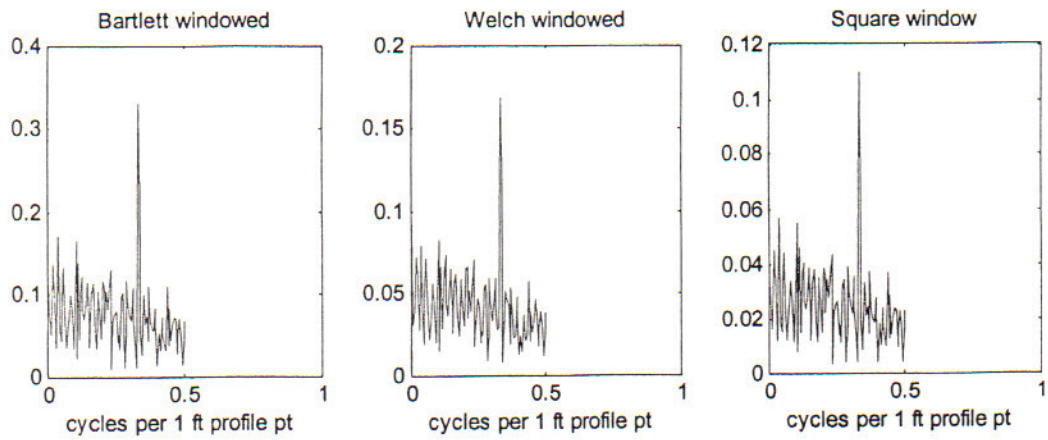


Figure 38 **Windowed Power Spectrums of 2ft Wood with 3' Spacing**

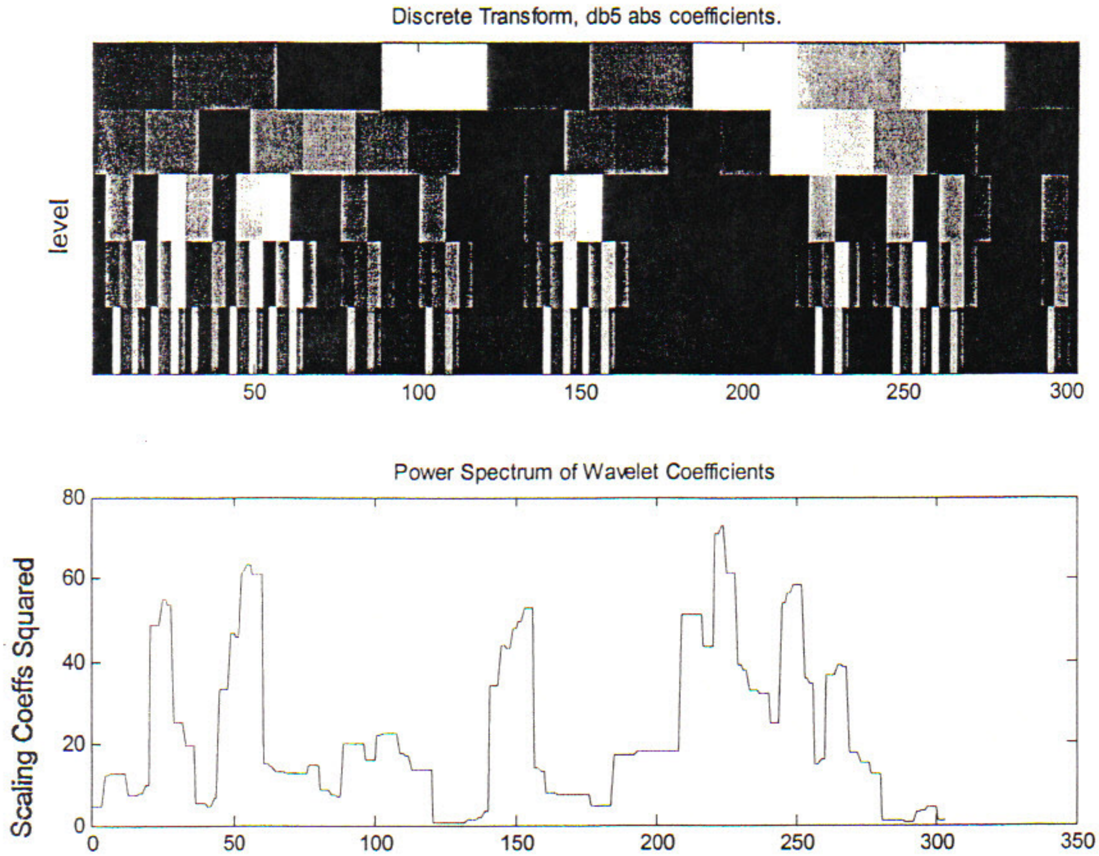


Figure 39 Wavelet Power Spectrum db5 Coefficients

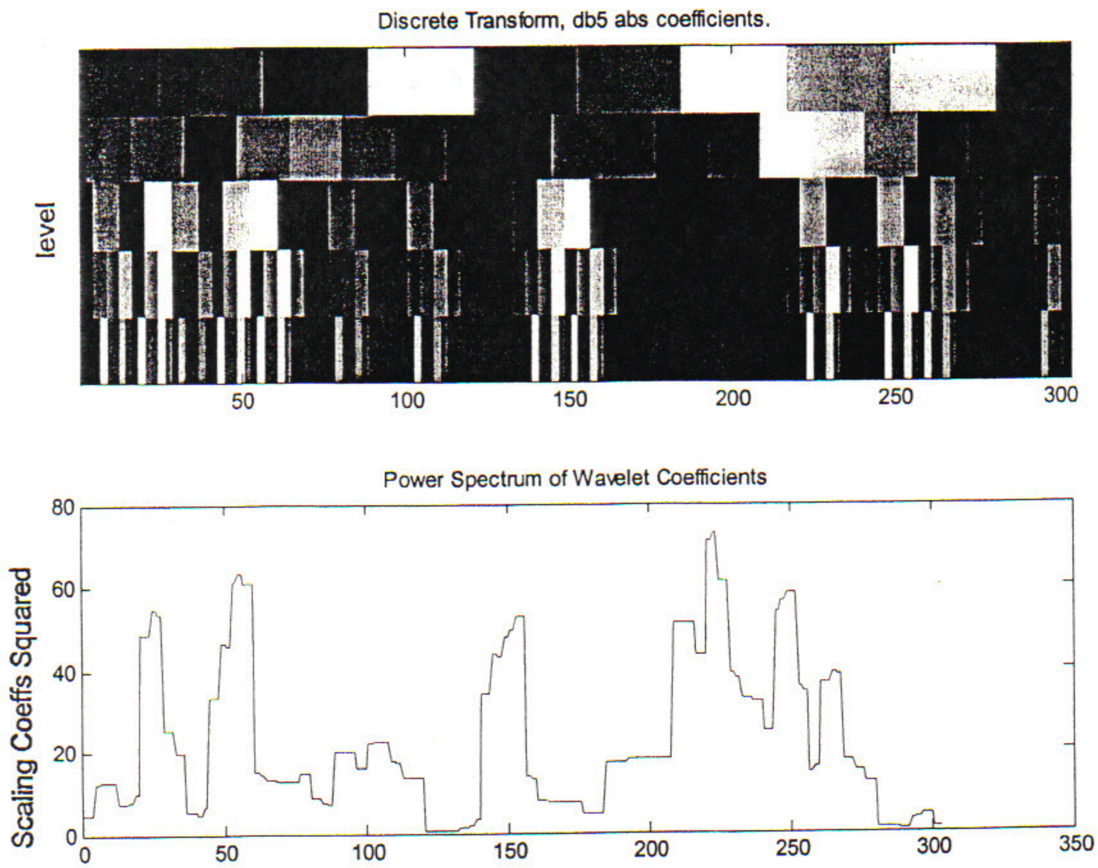


Figure 40 Wavelet Power Spectrum, db10 Coefficients

Conclusions

Examining the first set of graphs for each test course we see that choosing the correct scaling factor has a large effect on determining which of the various details in the transform show up in the display. Looking at figure 17 for the profile containing 4-inch obstacles at 2' spacing, the spikes in the transform that would characterize the profile varying greatly according to whether .1, .2, .3, .4,.5.,.6,.7,.8, or .9 of the frequency range is displayed. The same result is shown in figures 22,27,32,37 for the other four test courses that were analyzed. We must conclude therefore that the fourier transform by itself does not provide a very good way of differentiating the test courses by transformed power spectra.

It is likely that much of the cause for the different patterns in the spikes in the transformed power spectra is due to an "leakage" from nearby terms in the discrete frequency transforms. This is discussed in some detail in the reference Numerical Recipes by Press et al. Therefore it makes sense to examine various windowing techniques on the transform in order to modify the discrete equations in order to eliminate these artificial aliasing effects.

Examining the second in the set of graphs for each test course we see that windowing does eliminate most of this. And, choosing different windowing formulas seem to have little effect on the effectiveness of the formulas in differentiating graphs of the power spectrums of these urban test courses.

Finally looking at the last in the set of graphs for each of the test courses, we see that the wavelet coefficients of the test course power spectra also make a good means of differentiating the different courses. This is because the wavelet coefficients do not seem to be as dependent on the windowing scale used as the power spectra of the test course Fourier discrete transforms.

Comparing the two approaches; 1) windowed Fourier transforms and 2) Daubechie wavelets in each case we see that the number and magnitude of the frequency spikes in the windowed transforms are sometimes easier to use as a "quantification" of how to differentiate the test courses. And, sometimes the wavelet transforms are easier to use for this purpose.In figure 17 and 20 (4" wood inch obstacles at 2' spacing) the three spikes are easier to use as a quantification. In figure 22 and 26 (5' concrete at 1' spacing) the one spike in the windowed transforms is easier to use. In figures 28 and 32 (1" glass obstacles at 5' spacing) the wavelet transforms with three similar spikes are easier to use for classification. And, in figures 34 and 38 (8" steel obstacles at 4' spacing) the windowed transforms are unusable and the series of five irregular towers in the wavelet transforms seem easier to use as a means of differentiating the test course from others. Finally, in Figures 40 and 44 (3"wood at 2 ft spacing) the windowed Fourier transform with one clearly defined spike is the best method to use for differentiating the spectrum.

Recommendations

Table 1 below summarizes the results of the analysis for the test courses that were analyzed in this paper.

obstacles	spacing	scaled Fourier density	windowed Fourier density	wavelet
4" wood	2 ft	unusable	three similar sharp spikes	a series of irregular spikes
5" concrete	1 ft	unusable	one sharp spike	a series of irregular towers
8" steel	4 ft	unusable	unusable	five irregular towers
1" glass	5 ft	unusable	unusable	three similar spikes
3" wood	2 ft	unusable	one sharp spike	unusable

Table 1

It is recommended that test course profiles containing obstacles as above, which can be characterized by using windowed Fourier transforms and wavelet transforms, be used in ERDC vehicle testing for urban environments. By characterizing the test courses in this way, experimenters and testers will have a better understanding of how representative these courses are for vehicle testing in terms of test course profile power spectra. Also, it will be easier to decide, when specific vehicles having certain known specific responses to test course shock and vibration need new test course to be designed in order to better examine their performance in these types of environments.

It is instructive to compare the results of this mathematical analysis to characterize urban test sites with which the mobility division has recently developed to characterize linear vehicle test profiles (on and off-road) not containing obstacles. See the reference by Harrell [?] for the details of how this methodology works and its mathematical derivation. Its final output is two parameters, 1) a coefficient C which appears in the numerator of the spectral density function and provides a means of differentiating the course's spectral density 2) the dimension (possible fractal) of the spectral density curve which represents the inverse powers of the function representing the course's spectral density. In contrast to older methods, these two parameters do not depend on the detrending length used to smooth the elevation profile data and were shown to characterize the profiles analyzed. The results are shown in the Table below.

Table 3. Characterization of Two Road Sections and One Vehicle Test Course by the Spectral Characteristics of Their Elevation Profiles.

Course Name	Spectral Density Coefficient	Spectral Curve Inverse Dimension
MudLake Farming Road	1.2×10^{-3} $(\text{feet})^2$ cycle/foot	2
Forest Road	1.9×10^{-2} $(\text{feet})^2$ cycle/foot	2
Letourneau #1	1.3×10^{-1} $(\text{feet})^2$ cycle/foot	2

Briefly stated, these values were computed by detrending the course profile data using 3 different detrending lengths and plotting the results versus the RMS of the course which resulted. In the reference mentioned above, the MudLake Farming Road had a 10 foot detrended RMS of .3 and a 30 foot detrended RMS of .45 The Forest Road had a 10 foot detrended RMS of 1.21 and a 30 foot detrended RMS of 1.7.The Letourneau #1 course had 10 foot detrended RMS of 2.5 and a 30 foot detrended RMS of 4.3.

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I want to acknowledge Greg Green who wrote the first sections of this paper which describe, the VEHDYN simulation model, the test courses used for the vehicle simulations, and how the simulation models the power spectral response of the vehicles to the courses in this paper.

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Internet Sites

Rice University: www.dsp.rice.edu.
ATT Research Labs: www.wavelet.org.
Stanford University: www.stat.standford.edu/~wavelab

Appendix A

test course descriptions

```
 title                              filename
description
number of pts   spacing

DEBRISFIELD                      PWR4IN1FT000.dat
TYPE: 4-INCH,  HEIGHT:  4.0",  SPACING:  1',  ERROR:   0%
    2404    1.0000
DEBRISFIELD                      PWR4IN2FT025.dat
TYPE: 4-INCH,  HEIGHT:  4.0",  SPACING:  2',  ERROR:  25%
    2404    1.0000
DEBRISFIELD                      PWR4IN6FT050.dat
TYPE: 4-INCH,  HEIGHT:  4.0",  SPACING:  6',  ERROR:  50%
    2452    1.0000
DEBRISFIELD                      PWRC1025.dat
TYPE: CONCRETE,  HEIGHT:  5.0",  SPACING:  1',  ERROR:  25%
    2405    1.0000
DEBRISFIELD                      PWRG1FT00.dat
TYPE: GLASS,  HEIGHT:  1.0",  SPACING:  1',  ERROR:   0%
    2401    1.0000
DEBRISFIELD                      PWRG5FT075.dat
TYPE: GLASS,  HEIGHT:  1.0",  SPACING:  5',  ERROR:  75%
```

```
    2413     1.0000
DEBRISFIELD                          PWRS4FT025.dat
TYPE: STEEL,  HEIGHT:  8.0",  SPACING:  4',  ERROR:  25%
    2408     1.0000
DEBRISFIELD                          PWRS5FT000.dat
TYPE: STEEL,  HEIGHT:  8.0",  SPACING:  5',  ERROR:   0%
    2408     1.0000
DEBRISFIELD                          PWRW2FT100.dat
TYPE: WOOD,  HEIGHT:  3.0",  SPACING:  2',  ERROR: 100%
    2427     1.0000
DEBRISFIELD                          PWRW3FT00.dat
TYPE: WOOD,  HEIGHT:  3.0",  SPACING:  3',  ERROR:   0%
    2415     1.0000
```

Appendix B

MATLAB CODE

Program 1 Profile Plotting program

```
Matlab Program to compute plot course profiles along with 9 scaled power
spectrum plots in one figure

N=length(PWRW3FT00);
freq=(1:round(N/10));
height=PWRW3FT00(1:round(N/10));
plot(freq,height);
title('Profile Elevations 3-In Wood Obstacles 3ft spacing (1st part of
course)');
xlabel('feet');
ylabel('height in inches');
```

Program 2, computing the scaled fourier transform

```
% program t compute 10 fast fourier power spectrum transforms scaled
from
left to right on the x-axis.
```

The x-axis is scaled in terms of percent of Nyquist or basic fft sampling frequency

```
%name1=input('Enter input filename: ','s');
%load name1;
load PWRW3FT00.dat;
%name1='PWR4-IN1FT000.dat';
%cutoff=input('Enter lftmst freq cutoff (as percent of nyquist freq.):
');
height=PWRW3FT00(:,1);
Y=fft(height);
N=length(Y)
Y(1)=[];
nyquist=1/2;
for m=1:9
    cutoff=m*10
power=abs(Y(round(cutoff/100*(N/2)):N/2)).^2;
freq=(round(cutoff/100*(N/2)):N/2)/(N/2)*nyquist;
subplot(3,3,m);plot(freq,power);
%title('Power Spectrum');
% xlabel('cycles per second ');
%ylabel('Pwr Spectral Dens');

hold on;
end
xlabel('cycles per second ');
```

Program 3 to compute windowed transforms using overlapping data segments

```
function h=bartlett(j,a,b)
h=(1.0-abs(((j-1)-a)*b));  %bartlett window
end;

function h=welch(j,a,b)
h=(1.0-(((j-1-a)*b))^2);  %welch window
end;

function h=square(j,a,b)
h=1.0;  %square window
end;

% The program code below contains a feature in the code in which the
data is segmented and either overlapped or not. As explained in the
reference by Press and Vettering on Numerical Computing, using overlaps
in segmented data can be useful in situations in which the smallest
spectral variance per computer operation is desired. For characterizing
obstacle courses with equally spaced geometrical obstacles it is
advisable not use overlapping in computing the power spectrum in order
to avoid mixing the geometry of the segments together on the overlaps.

function pspectrum(k,ovrlap)
load PWRW3FT00.dat;
w2=PWRW3FT00(:,1);
N=length(w2)
if(ovrlap)
m=round(length(w2)/(2*k+1))
```

```
else
m=round(length(w2)/(4*k))-1
end;
for r=1:4*(m+1)
     w1(r)=0;
     w8(r)=0;
     w88(r)=0;
end;
for r=1:m w2(r)=0;
end;
mm=m+m; den=0.0;   dden=0;ddden=0;
m4=mm+mm; m43=m4+3;
m44=m43+1;
facm=m;
facp=1.0/m;
sumw=0;wsumw=0;wwsumw=0;
den=0;dden=0;ddden=0;
for j=1:mm
     sumw=sumw+(bartlett(j,facm,facp))^2;
     wsumw=wsumw+(welch(j,facm,facp))^2;
     wwsumw=wwsumw+(square(j,facm,facp))^2;
end;

for j=1:mm
     p(j)=0.0;p2(j)=0;p3(j)=0;
end;
     if(ovrlap)
      for j=1:m
           w2(j)=PWRW3FT00(j,1);
      end;
      N=m;
     end;

for kk=1:k
     for joff=-1:0
          if(ovrlap)
               for j=1:m
                    w1(joff+j+j)=w2(j);
               end;
               for j=1:m
                    w2(j)=PWRW3FT00(N+j,1);
               end;
               joffn=joff+mm;
              for j=1:m
                  w1(joffn+j+j)=w2(j);
              end;
          else
               for j=joff+2:2:4*m
                    w1(j)=PWRW3FT00(j,1);
               end;
end;
end;

for j=1:mm
     j2=j+j;
     w=bartlett(j,facm,facp);
     ww=welch(j,facm,facp);
     www=square(j,facm,facp);
     w1(j2)=w1(j2)*w;
     w1(j2-1)=w1(j2-1)*w;
     w8(j2)=w1(j2)*ww;
```

```
     w8(j2-1)=w1(j2-1)*ww;
      w88(j2)=w1(j2)*www;
     w88(j2-1)=w1(j2-1)*www;
end;
p=fft(w1);
p2=fft(w8);
p3=fft(w88);
     p(1)=p(1)+(w1(1)^2+w1(2)^2);
     p2(1)=p2(1)+(w8(1)^2+w8(2)^2);
     p3(1)=p3(1)+(w88(1)^2+w88(2)^2);
     for j=2:m
         j2=j+j;
        p(j)=p(j)+(w1(j2)^2+w1(j2-1)^2+w1(m44-j2)^2+w1(m43-j2)^2);
    end;
    for j=2:m
         j2=j+j;
        p2(j)=p2(j)+(w8(j2)^2+w8(j2-1)^2+w8(m44-j2)^2+w8(m43-j2)^2);
    end;
    for j=2:m
         j2=j+j;
        p3(j)=p3(j)+(w88(j2)^2+w88(j2-1)^2+w88(m44-j2)^2+w88(m43-j2)^2);
    end;
    den=den+sumw;
    dden=dden+wsumw;
    ddden=ddden+wwsumw;
end;
for j=1:m p(j)=p(j)/den;p2(j)=p2(j)/dden;p3(j)=p3(j)/ddden;
end;
     p(1)=[];p2(1)=[];p3(1)=[];
nyquist=1/2
power=abs(p(1:round(m/2)));
power2=abs(p2(1:round(m/2)));
power3=abs(p3(1:round(m/2)));
freq=(1:round(m/2))/(m/2)*nyquist;
hold on;
subplot(2,3,1);plot(freq,power);
title('Bartlett windowed');
xlabel('cycles per 1 ft profile pt ');
subplot(2,3,2);plot(freq,power2);
title('Welch windowed');
xlabel('cycles per 1 ft profile pt ');
subplot(2,3,3);plot(freq,power3);
title('Square window');
xlabel('cycles per 1 ft profile pt ');

end;
```

Program 4 to compute wavelet transforms

```
% Mathlab code to compute wavelet power spectrums
% First, the discrete wavelet transform is plotted
% Then the squared sum of scaled coefficients at each
% spatial point is plotted. This is a spectral estimator
% called the 'time marginal', analagously defined to
% the Welch estimator. cf. paper by Abry
function wvltpspectrum(level,wvlet)
% wvlet type 1= Daubauchies level 5 2=Daubachies level 10
name1=input('Enter input filename: ','s');
A=load(name1);
w2=A(:,1);
```